PRESSURE IN SLIDE JOURNAL BEARING FOR LAMINAR, UNSTEADY LUBRICATION BY VARIABLE VISCOSITY OIL

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Abstract

Present paper shows the results of numerical solution of laminar, unsteady lubricated cylindrical slide bearing. Laminar, unsteady oil flow is performed during periodic and unperiodic perturbations of bearing load or is caused by the changes of gap height in the time. Above perturbations occur during the starting and stopping of machine. The solutions apply to infinite length of lubricated Newtonian oil with dynamic viscosity depends on pressure. The disturbances related with unsteady velocity oils on the journal and on the sleeve. The results shown on diagrams of hydrodynamic pressure in dimensionless form in time intervals of displacement duration.

1. Introduction

This article refers to the unsteady, laminar flows issue, in which modified Reynolds number $Re^{*}=Re\psi$ is smaller or equal to 2. This flows [4], [5] are also determined by Taylor number $Ty=Re\sqrt{\psi}$ which is smaller or equal to 41.1. Increasing of criterial numbers causes firstly conversion into unsteady laminar-turbulent flows and later conversion into turbulent flows [1]. Laminar, unsteady oil flow is performed during periodic and unperiodic perturbations of bearing load or is caused by the changes of gap height in the time. Above perturbations occur mostly during the starting and stopping of machine. Lubricated oil disturbance velocity the pin and on the bearing shell was also consider in the article. Reynolds equation system describing Newtonian oil flow in the gap of transversal slide bearing was discussed in the articles [3], [4]. Mentioned equations were used in this article. Velocity perturbations of oil-lubricated flow on the pin can be caused by torsion pin vibrations during the rotary movement of the shaft. Perturbations are proportional to torsion vibration amplitude, frequent constraint and to pin radius of the shaft. Oil velocity perturbations on the shell surface can be caused by rotary vibration of the shell together with bearing casing. This movement can be considering as kinematic constraint for whole bearing friction node. Isothermal bearing model can be approximate to bearing operation in friction node under steady-state thermal load conditions for example bearing in generating set on ship. In bearing calculation operating in pressure of the order of 10 MPa, dynamic viscosity change from pressure was taken into consideration.

2. Modified Reynolds Equation

The unsteady, laminar and isotherm flow Newtonian oil in journal bearing gap is described for modified Reynolds equation [1], [2] from newtonian oil with constant and variable dynamic viscosity depended for pressure. In considered model we assume small unsteady disturbances and in order to maintain the laminar flow, oil velocity V_i^{*} and pressure

p_1^* are total of dependent quantities $\widetilde{V}_i$; $\widetilde{p}_1$ and independent quantities $V_i; p_1$ from time [3], [5] according to equation (1).

$$V_i^* = V_i + \widetilde{V}_i, \qquad i=1, 2, 3,$$
$$p_1^* = p_1 + \widetilde{p}. \tag{1}$$

Unsteady components of dimensionless oil velocity and pressure we [4] in following form of infinite series:

$$\widetilde{V}_i(\varphi; r_1; z_1; t_1) = \sum_{k=1}^{\infty} V_i^{(k)}(\varphi; r_1; z_1)\exp(jk\omega_0 t_0 t_1),$$
$$\widetilde{p}_1(\varphi; z_1; t_1) = \sum_{k=1}^{\infty} p_1^{(k)}(\varphi; z_1)\exp(jk\omega_0 t_0 t_1), \tag{2}$$

where: i=1,2,3 ; ω_0 – angular velocity perturbations in unsteady flow;
$j=\sqrt{-1}$ - imaginary unit.

Reynolds equation describing total dimensionless pressure p_1^* (sum steady and unsteady components) in oil journal bearing gap [1] by unsteady, laminar, isotherm Newtonian flow along with disturbances of peripheral velocity V_{10} on the journal and V_{1h} on the sleeve and disturbances of velocity on journal length V_{30} on the journal and V_{3h} on the sleeve has following form:

$$\frac{\partial}{\partial\varphi}\left\{\frac{(h_1)^3}{e^{Kp_1}}\left[\frac{\partial p_1^*}{\partial\varphi} - K\left(p_1^* - p_1\right)\frac{\partial p_1}{\partial\varphi}\right]\right\} + \frac{1}{L_1^2}\frac{\partial}{\partial z_1}\left\{\frac{(h_1)^3}{e^{Kp_1}}\left[\frac{\partial p_1^*}{\partial z_1} - K\left(p_1^* - p_1\right)\frac{\partial p_1}{\partial z_1}\right]\right\} =$$
$$= 6\frac{\partial h_1}{\partial\varphi} + \frac{1}{2}\frac{\rho_1}{\eta_1}\mathrm{Re}\,Str\,\omega_0 t_0\left\{\frac{\partial}{\partial\varphi}\left[h_1^3\left(V_{10} + V_{1h}\right)\right] + \frac{1}{L_1^2}\frac{\partial}{\partial z_1}\left[h_1^3\left(V_{30} + V_{3h}\right)\right]\right\}\sum_{k=1}^{\infty}A_k + \tag{3}$$
$$-6\left\{\frac{\partial}{\partial\varphi}\left[h_1\left(V_{10} + V_{1h}\right)\right] + \frac{1}{L_1^2}\frac{\partial}{\partial z_1}\left[h_1\left(V_{30} + V_{3h}\right)\right] - 2\left(V_{1h}\frac{\partial h_1}{\partial\varphi} + \frac{1}{L_1^2}V_{3h}\frac{\partial h_1}{\partial z_1}\right)\right\}\sum_{k=1}^{\infty}B_k$$

where: $0 \le \varphi \le \varphi_e$; $0 \le r_1 \le h_{p1}$; $-1 \le z_1 \le 1$; $0 \le t_1 \le t_k$; $p_1^* = p_1^*(\varphi; z_1; t_1)$.

Dynamic oil viscosity η is depended on pressure by Barrus formula [5] and has following form:

$$\eta = \eta_0 e^{\alpha(p-p_a)} \approx \eta_0 e^{\alpha p} = \eta_0\eta_1 , \tag{4}$$

where: η_o- the dynamic oil viscosity for atmospheric pressure $p = p_a \approx 0$, η – the dynamic oil viscosity function, α – the pressure influence piesocoefficient of the oil viscosity, η_1 – dimensionless dynamic viscosity depending on pressure $\eta_1=\exp(\alpha p)$.

Components of oil velocity V_φ, V_r, V_z in cylindrical co-ordinates r,φ,z have presented as V_1, V_2, V_3 in dimensionless form:

$$V_\varphi = UV_1{}^*, \qquad V_r = \psi UV_2{}^*, \qquad V_z = \frac{U}{L_1} V_3^*, \tag{5}$$

where: U – peripheral journal velocity U=ωR; ω – angular journal velocity; R – radius of the journal; ψ– dimensionless radial clearance $(10^{-4} \le \psi \le 10^{-3})$; L_1 – dimensionless bearing length:

$$\psi = \frac{\varepsilon}{R}, \qquad L_1 = \frac{b}{R}, \tag{6}$$

where: b – length of the journal; ε – radial clearance.

Putting following quantities: dimensionless values density ρ_1, hydrodynamic pressure p_1^*, dynamic oil viscosity η_1, time t_1, longitudinal gap height h_1, radial co-ordinate r_1

$$\eta = \eta_0 \eta_1, \quad \rho = \rho_0 \rho_1, \quad z = b z_1, \quad h = \varepsilon h_1,$$
$$t = t_0 t_1, \quad r = R(1 + \psi r_1), \quad p = p_0 p_1^*, \quad K = \alpha p_0, \quad p_0 = \frac{U\eta_0}{\psi^2 R^2}. \tag{7}$$

Rule of putting dimensionless velocity and pressure quantities in unsteady and steady part of the flow stays similar. Following symbols with bottom zero index signify density, dynamic viscosity, pressure and time describe characteristic dimension values assigned to adequate quantities. Parameter K characterizes dimensionless oil dynamic viscosity change from pressure. Laminar, unsteady, unsymmetrical oil flow is characterized in equation (3) with two criterial numbers: Re Reynolds Number, Str Strouhal Number:

$$\mathrm{Re} = \frac{U\rho_0 \varepsilon}{\eta_0}, \qquad \mathrm{Str} = \frac{\varepsilon}{U t_0}. \tag{8}$$

Dynamic viscosity of oil is depended on pressure and presented as sum from steady part (not dependent from time) and unsteady (dependent from time) in dimensionless form [1]:

$$\eta_1^* = \eta_1 + \tilde{\eta}_1, \qquad \eta_1 = e^{Kp_1}, \qquad \tilde{\eta}_1 = K\tilde{p}_1\eta_1. \tag{9}$$

For circumference unsteady components of oil velocity (2) we assume on the movable journal by $r_1=0$ and in the motionless sleeve by $r_1=h_1$ surface:

$$\begin{aligned} V_1^{(k)}(\varphi; r_1 = 0; z_1) &= \frac{1}{k^2} V_{10}, \\ V_1^{(k)}(\varphi; r_1 = h_1; z_1; t_1) &= \frac{1}{k^2} V_{1h}, \end{aligned} \tag{10}$$

where: V_{10}- dimensionless function for velocity of perturbation in direction φ on the journal and V_{1h} on the sleeve.

For longitudinal unsteady components of oil velocity we assume on the oscillating journal $r_1=0$ and sleeve $r_1=h_1$ surface the following boundary conditions:

$$\begin{aligned} V_3^{(k)}(\varphi; r_1 = 0; z_1; t_1) &= \frac{1}{k^2} V_{30}, \\ V_3^{(k)}(\varphi; r_1 = h_1; z_1; t_1) &= \frac{1}{k^2} V_{3h}, \end{aligned} \tag{11}$$

where: V_{30}- dimensionless function for velocity of perturbation in direction z_1 on the journal and V_{3h} on the sleeve.

Sum for series in right side Reynolds equation (3) to result from conservation of the momentum solutions and has the following form:

$$\sum_{k=1}^{\infty} A_k = \sum_{k=1}^{\infty} \frac{\sin(k\omega_0 t_0 t_1)}{k} = \begin{cases} \dfrac{\pi - \omega_0 t_0 t_1}{2} & 0 < t_1 < 1, \\ 0 & t_1 = 0;\ 1, \end{cases}$$
$$\sum_{k=1}^{\infty} B_k = \sum_{k=1}^{\infty} \frac{\cos(k\omega_0 t_0 t_1)}{k^2} = \frac{1}{4}\left[(\pi - \omega_0 t_0 t_1)^2 - \frac{\pi^2}{3}\right] \quad 0 \le t_1 \le 1. \tag{12}$$

3. Hydrodynamic Pressure

Further analyze consider bearing with infinity length and it is assumed that disturbance velocity does not depend on cylindrical co-ordinate φ. After double integral in 'term of variable φ and after imposing edge condition equation (3) determine total dimensionless hydrodynamic pressure function in following form:

$$p_1^*(\varphi) = \frac{1}{1 - Kp_{10}}\left[-\frac{1 - Kp_{10}}{K} \ln\left|1 - Kp_{10}\right| - p_{10}(V_{10} - V_{1h})\sum_{k=1}^{\infty} B_k + \right.$$
$$\left. + \frac{1}{2}\rho_1 \operatorname{Re}^* n(V_{10} + V_{1h})\left(\varphi - h_{1e}^3 \int_0^{\varphi} \frac{d\varphi}{h_1^3} - K\int_0^{\varphi} p_{10} d\varphi\right)\sum_{k=1}^{\infty} A_k \right] \tag{13}$$

for $\quad 0 \le \varphi \le \varphi_e\ ; \quad 0 \le t_1 \le t_k\ ;\ p_1^* = p_1^*(\varphi; t_1)$.

Pressure p_{10} is located in the oil gap by steady flow and by constant oil dynamic viscosity independent from pressure (K=0):

$$p_1(\varphi) = 6\int_0^{\varphi} \frac{h_1(\varphi) - h_{1e}}{h_1^3(\varphi)} d\varphi \quad \text{for } 0 \le \varphi \le \varphi_e. \tag{14}$$

Dimensionless total pressure by disturb flow and by constant oil dynamic viscosity independent from pressure (K=0):

$$p_1^*(\varphi) = p_{10} + \frac{1}{2}\rho_1 \operatorname{Re}^* n(V_{10} + V_{1h})\left(\varphi - h_{1e}^3\int_0^{\varphi}\frac{d\varphi}{h_1^3}\right)\sum_{k=1}^{\infty} A_k - p_{10}(V_{10} - V_{1h} \sum_{k=1}^{\infty} B_k. \tag{15}$$

Disturbance pressure in unsteady flow part can be presented with common formula for constant and variable dynamic viscosity $(K \ge 0)$:

$$\tilde{p}_1 = \frac{1}{1 - Kp_{10}}\left[\frac{1}{2}\rho_1 \operatorname{Re}^* n(V_{10} + V_{1h})\left(\varphi - h_{1e}^3\int_0^{\varphi}\frac{d\varphi}{h_1^3} - K\int_0^{\varphi} p_{10} d\varphi\right)\sum_{k=1}^{\infty} A_k - p_{10}(V_{10} - V_{1h})\sum_{k=1}^{\infty} B_k\right]. \tag{16}$$

Dimensionless pressure in steady and unsteady flow part at the film beginning φ=0 and at the film end φ= φ_e assume values equal to atmospheric pressure. At the film end extra edge condition is being fulfilled (pressure derivative by the angular co-ordinate φ resets). Formula

(13) was used in numerical way to determine co-ordinate φ_e describing oil the film end position. In numerical analyze of pressure distribution values in formula (13) extra expression consisting of criteria numbers is presented below:

$$\rho_1 n \mathrm{Re}^* = \rho_1 \mathrm{Re}\, Str \omega_0 t_0 . \tag{17}$$

where: Re^* - modified Reynolds Number $\mathrm{Re}^* \equiv \psi \mathrm{Re}$, $n=\omega_0/\omega$ multiplication factor determined frequency periodical perturbations and frequency of journal rotation ω.

In disturbances of peripheral velocity case caused by journal bearing torsion vibrations of main engine, n value is equal to number of cylinder c in two-stroke engine or in four-stroke engine to number of cylinders c/2.

4. Numerical Results

In numerical calculation example oil with constant density was assume, what is equivalent to quantity ρ_1. In presented calculation way an expression value is assumed $n\rho_1\mathrm{Re}^* = 12$, what is approximately equivalent to force over first frequency torsion vibrations force of six cylinder engine shaft. This takes place by laminar unsteady flow. Time of reference t_o is a period of velocity disturbances dispersion. Dimensionless oil gap height for bearing dependent eccentricity λ is described as follows:

$$h_1 = 1 + \lambda \cos\varphi . \tag{18}$$

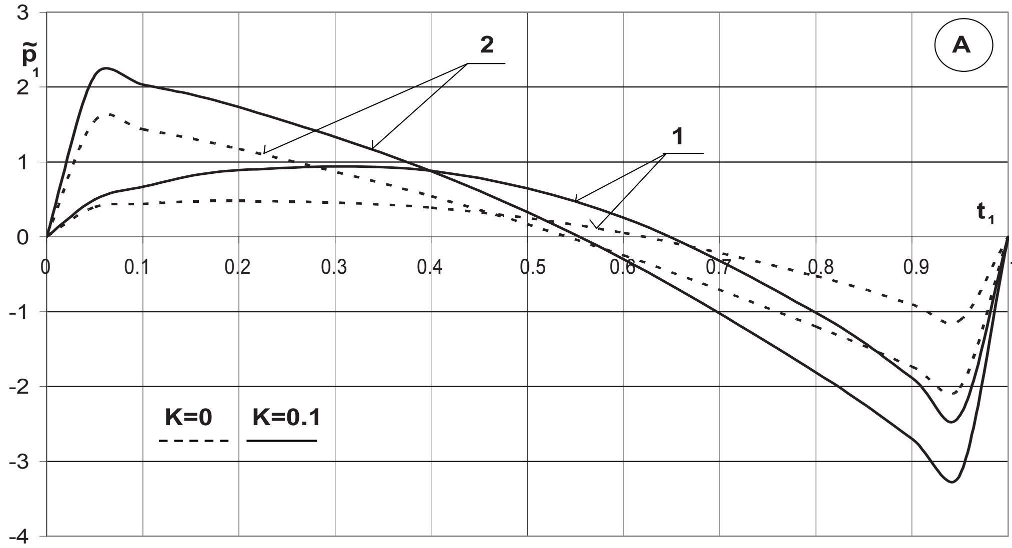
In numerical calculations influence of velocity disturbances on the journal and on the sleeve were analyze. Examples apply to bearing with constant dependent eccentricity $\lambda=0.6$. Pressure distribution by wrapping angle and pressure distribution in time function at selected point at the journal surface. Numerical calculation results are presented by following tangential velocity perturbations:

1. velocity perturbations on the journal $V_{10}=0.05$,
2. velocity perturbations on the journal $V_{10}=0.05$ and on the sleeve $V_{1h}=0.025$,
3. velocity perturbations on the journal $V_{10}=0.05$ and on the sleeve $V_{1h}=0.05$,
4. velocity perturbations on the journal $V_{10}=0.05$ and on the sleeve $V_{1h}= -0.05$.

Unsteady pressure is changing due velocity perturbations time and it is in function of time and position on the journal. It is a periodic function with the following lasting period of velocity perturbation. Pressure perturbation course in point $\varphi=145$ on the journal surface in dimensionless time function in case of velocity perturbation on the journal and on the sleeve is presented on fig. 1A (perturbation 1 and 2) and fig. 1B (perturbations 3 and 4).

Above graphs are made for constant viscosity and for viscosity in dependence on pressure where K=0.1.When oil velocity perturbations on the journal are compatible to journal tangential velocity than perturbation pressure increase otherwise the pressure is decreasing. In this case decrease considerably bigger than increase and it last shorter than half of perturbation period.

In case of velocity perturbation on the sleeve it is opposite. There is a lack of graphs for this example. Periods of pressure increase and decrease are non-symmetrical in case of different perturbation velocity values (graph 2). When perturbations velocity values are equal and directions are the same or opposite then perturbation pressure is symmetrical in time (graph 3 and 4,fig.1). Pressure perturbation distribution by wrapping angle is changing in time, giving in different time periods maximal or minimal pressure.



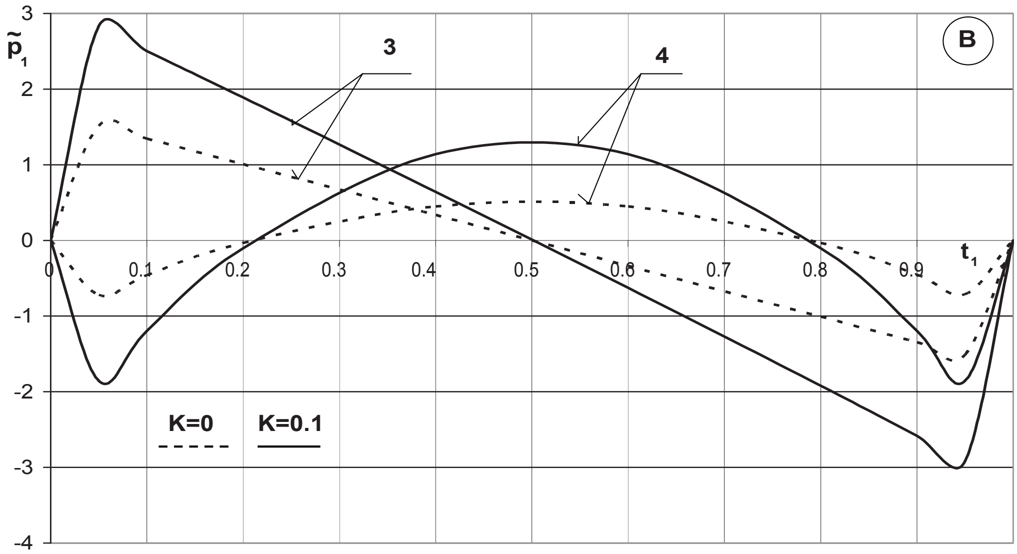


Fig. 1. Pressure distributions $\widetilde{p}_1$ in place $\varphi=145°$ in the time t_1 for constant oil viscosity (K=0) and for oil viscosity in dependence on pressure (K=0.1) by velocity perturbations: 1) $V_{10}=0.05$; $V_{1h}=0$; 2) $V_{10}=0.05$; $V_{1h}=0.025$; 3) $V_{10}=0.05$; $V_{1h}=0.05$; 4) $V_{10}=0.05$; $V_{1h}=-0.05$

Maximal and minimal pressure distribution for in considered velocity perturbation examples are presented on fig. 2A and 2B. In order to compare influence of viscosity variable in dependence on pressure (graph b), pressure distribution for oil with constant viscosity (K=0), which is independent from pressure, were plotted (graph a). When viscosity is in dependence of pressure is causes an increase of steady pressure and perturbation pressure on both maximal and minimal pressure sides. Steady pressure flow sum up with perturbation pressure and total distribution of maximal and minimal pressure by bearing wrapping angle is received. This is the border pressure distribution for given type of perturbation.

At the fig. 3A and 3B maximal and in minimal total pressure distribution are presented for considered examples of velocity perturbation marked as in fig. 1 and 2. Oil lubricated bearing with oil viscosity in dependence on pressure (K=0.1), with bigger pressure reserve respond to velocity perturbation despite higher pressure perturbation than in case when lubricated with oil with independent viscosity from pressure (K=0). This pressure reserve is result from stationary pressure increase by oil lubrication with oil viscosity variable in dependence on pressure.

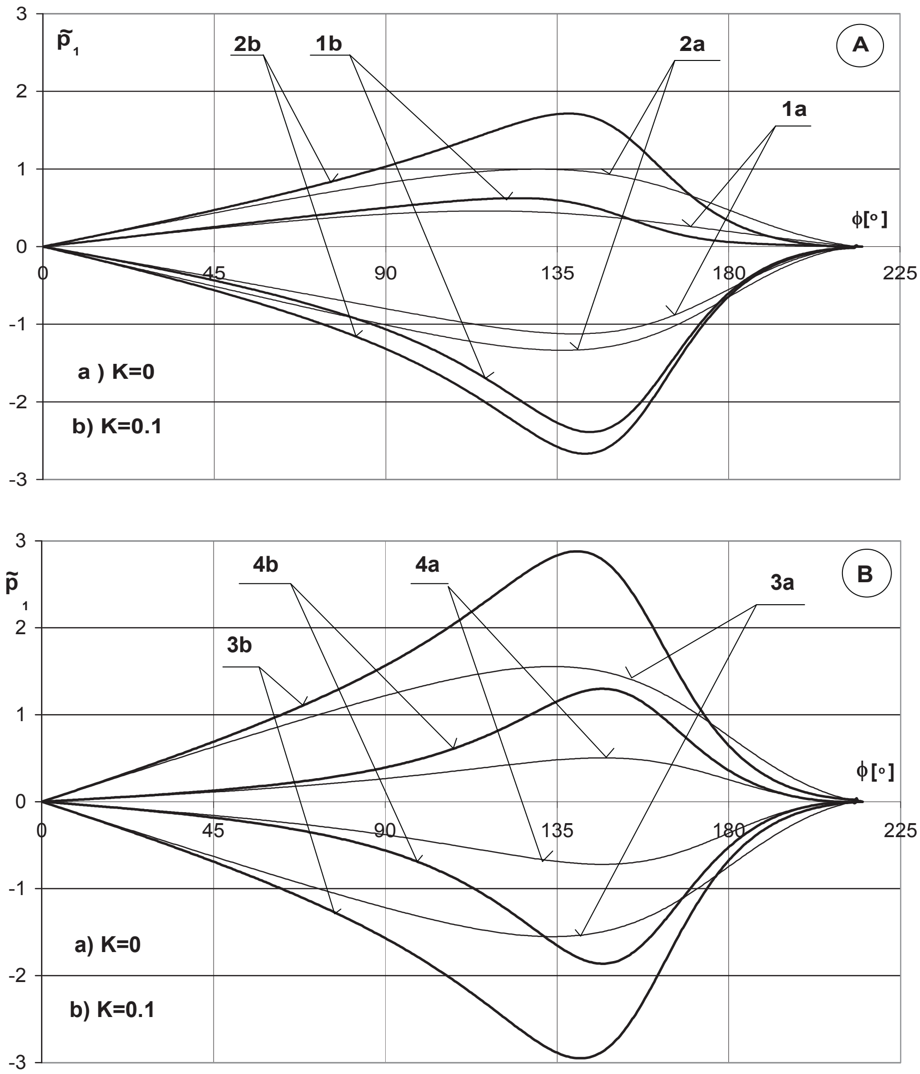


Fig. 2. Unsteady part maximal and minimal pressure distributions $\widetilde{p}_1$ in direction φ a) for constant oil viscosity (K=0), b) for oil viscosity in dependence on pressure (K=0.1) by velocity perturbations: 1) V_{10}=0.05; V_{1h}=0; 2) V_{10}=0.05; V_{1h}=0.025. 3) V_{10}=0.05; V_{1h}=0.05;4) V_{10}=0.05; V_{1h}= - 0.05

5. Conclusions

Presented Reynolds Equation solution for unsteady, laminar, Newtonian flow of lubricated oil to enable initial opinion to hydrodynamic pressure distribution as a basic slide bearing operating parameter. Unsteady velocity perturbation on the journal and sleeve effect on hydrodynamic pressure distribution in lubricated gap. Influence of both perturbations is quantitative different and always bigger when oil viscosity depend on pressure. Pressure variations in bearing have periodical character equal to periodical velocity perturbation time and this variations value and character depend on type of perturbation. Author is aware of simplifications that were assumed in presented model which apply to Newtonian oil and to isothermal bearing model.

Despite that presented calculation example apply to bearing with infinity length, obtained conclusions can be useful to pressure distribution and aerodynamic lift assessment by laminar, unsteady lubrication of cylindrical slide bearing with finite length.

Presented results can be also useful as a comparison values in numerical modelling of laminar, unsteady, non-Newtonian fluids in lubricated, lubricated crosswise gap of cylindrical slide bearing.

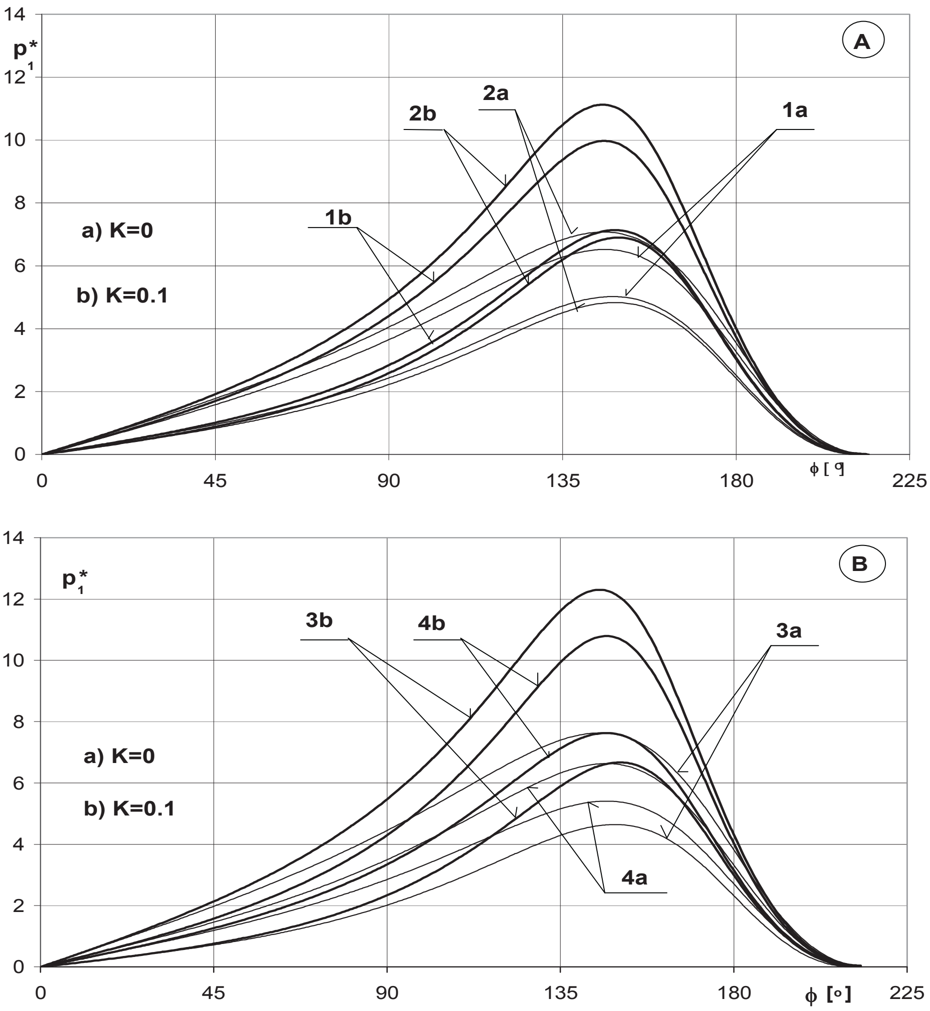


Fig. 3. Total maximal and minimal pressure distributions p_1^* in direction φ a) for constant oil viscosity (K=0) and b) for oil viscosity in dependence on pressure (K=0.1) by velocity perturbations: 1) $V_{10}=0.05$; $V_{1h}=0$ 2) $V_{10}=0.05$; $V_{1h}=0.025$. 3) $V_{10}=0.05$; $V_{1h}=0.05$; 4) $V_{10}=0.05$; $V_{1h}=-0.05$

References

[1] Krasowski P., Laminar, unsteady lubrication of slide journal bearing in magnetic field with dynamic viscosity depended on pressure, (in Polish),Tribologia 4/2002 (184), pp. 1189-1200, 2002.

[2] Krasowski P., Modelling of laminar unsteady and unsymmetrical oil flow in slide journal bearing gap. Tribologia 5/2002, pp. 1425-1436, 2002.

[3] Teipel I., Waterstraat A., The Reynolds equation for lubrication under unsteady conditions., Proceedings The IX Canadian Congress of Applied Mechanics, University of Saskatchewan, 497-502, 1983.

[4] Wierzcholski K., Mathematical methods in hydrodynamic theory of lubrication, Technical University Press, Szczecin 1993.

[5] Wierzcholski K., Theory unconventional lubricant slide journal bearing, (in Polish),Technical University Press, Szczecin 1995.

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